Period ________

Modeling Soft-Drink Packaging

Name ____________________

Score ______/ 35

Group Members

Staple the Rubric on Top of your Project

____

____

Criteria	Exemplary (8)	Acceptable (5-7)	Poor (2-4)	Unacceptable (1)
Poster	• Mathematics is correct with only a very minor flaw (not having to do with the main idea of the problem.)	• You miss one thing within the mathematics content of your presentation.	• Show very little content knowledge.	• Do not complete the poster portion of the project.
Verbal Models	• Provides highly detailed verbal models for all methods • Shows understanding of volume and surface area formulas	• Provides adequately detailed verbal models for all methods • Shows understanding of volume and surface area formulas	• Poorly Detailed verbal models. • Some verbal models may be missing	• Little effort to make Verbal Models • Verbal Models almost non-existent
Solutions	• Provides highly detailed, accurate, and neat Solution for all methods • Shows detailed work for finding solutions • Work shows an understanding of surface area and volume and how they can be used in real life.	• Provides adequately detailed Solutions • Important formulas and work shown • Work shows an understanding of surface area and volume and how they can be used in real life	• Poorly Detailed and inaccurate solutions • Essential work is missing or inaccurate • Steps to solution are missing.	• Little effort to find solutions • Solutions and work almost non-existent
Summary Questions	• Provides highly detailed and insightful explanations of Summary questions • Uses complete sentences when answering questions	• Provides adequately detailed explanations of summary questions. • Uses complete sentences when answering questions	• Poorly Detailed explanation of summary questions • Does not use complete sentences when answering questions	• Little effort to make to explain reasoning • Summary almost non-existent

Sub-Totals ___ + ________ + ________ + ________ + ________

Cooperation 3

[redacted]

Modeling Soft-Drink Packaging

All students must turn in a completed packet!

In this activity you will use mathematical modeling to describe and improve the efficiency of secondary soft-drink packaging. The primary packaging is the can; the secondary packaging is the container that holds several cans.

Before beginning the modeling process, consider the impact that an improvement in the design of soft-drink packages might have. Because the volume of soft drinks consumed is large, even a small savings on the cost of a secondary package could mean a lot to the soft-drink industry.

1. According to the National Soft Drink Association, 62.6 billion cans of soft drinks were consumed in the United States in 1995. Suppose that you find a way to save the soft-drink industry one-tenth of a cent ($.001) on the packaging of each twelve-pack sold.
   a) Estimate the total annual savings to the soft-drink industry. Show all your work and use a complete sentence in your answer.

   62,600,000,000 x .001 = 62600000
   The total annual savings to the soft drink industry would be around $62,600,000.

   b) If you receive the royalties worth 10 percent of the savings to the industry, estimate your annual income for the use of your innovation in the United States. Show all your work and use a complete sentence in your answer.

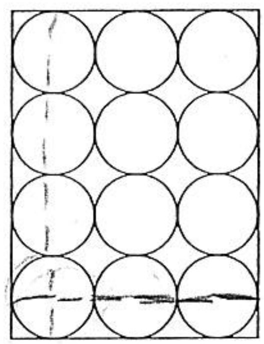
   62,600,000 x .1 = 6,260,000
   You would receive $6,260,000 annually as a ten-percent royalty payment for your innovation.

[redacted]

Modelers use various mathematical tools to reach conclusions. Questions 3 and 4 show how geometric formulas can help calculate packaging efficiency. Questions 3 and 4 also introduce simplification, a necessary part of the modeling process.

2. Mathematical modeling always involves simplification. Simplification is often done to eliminate relativity unimportant information, but it is also done for convenience. The figure shown is a two-dimensional representation of the twelve-pack commonly used for secondary packaging of soft drinks.

$\frac{385.8432}{491.52} = \frac{78.5}{100}$



$3.14(3.2^2) = 32.1536$

$32.1536 \times 12 = 385.8432$

$3.2 \times 6 = 19.2$

$3.2 \times 8 = 25.6$

$25.6 \times 19.2 = 491.52$

a) The radius of a soft drink can is approximately 3.2 centimeters. Determine how well the cans use the space in this two-dimensional package, that is, calculate the percent of package space used by the cans. Show all your work and use a complete sentence in your answer.

The total 2-D area occupied by the cans is 385.8432 which is equivalent to 78.5% of the box.

b) Show that your answer is independent of the size of the cans, that is, repeat your work, but use ***r*** to represent the radius.

$2r(3) \times 2r(4)$

$6r \times 8r$

$48r^2$

$\pi r^2 \times 12$

$37.68r^2$ ← used

$48r^2 - 37.68r^2 = 10.32r^2$ unused

$\frac{37.68}{48}$ $\frac{78.5}{100}$

↖ This ratio verifies that 78.5% will be used not dependent on the radius

c) Show that simplification to two dimensions does not affect the result, that is, find the volume of the package and the volume of the cans in terms of the radius ***r*** and height ***h*** of the cans, then calculate the percent of space used by the cans.

$48r^2h$

↑ volume of total box

$3.14(r^2)(h)(12)$

$37.68(r^2)(h)$ ← used space

$\frac{37.68}{48} = \frac{78.5}{100}$

The used space of the 3-D object is the same space as of the 2-D object 78.5%

3. Determine the amount of packaging material used by the standard twelve-pack, as illustrated in the figure. For simplicity, assume that the material has no thickness and use the area of the packaging material as your measure. Also assume that the package requires no overlap of packaging material. Note that the height of a soft-drink can is about 12 centimeters. Show all your work and use a complete sentence in your answer.

$S = 2B + Ph$ $\quad 2(25.6 \times 19.2) + (51.2 + 38.4)12$

$2(491.52) + 89.6(12)$

$983.04 + 1075.2$

2058.24

The total surface area of a standard twelve pack would be 2058.24 cm².

Suppose that a competing design uses 1500 square centimeters of packaging material to hold eight cans. Is this design more efficient than the standard twelve-pack? Explain.

$\frac{1500}{8}$ ~~18000~~ ~~96~~ 187.5 cm² packaging per can

$\frac{2058.24}{12}$ ~~16465.92~~ ~~96~~ 171.52 cm² packaging per can

15.98

The 12-pack packaging is more effective because it uses 15.98 cm² less packaging per can.

You have assessed the efficiency of a standard twelve-pack by two measures: the percent of package space used by the cans and the packaging material used per can. In the first case, the modeling objective is maximization; in the second case, the modeling objective is minimization.

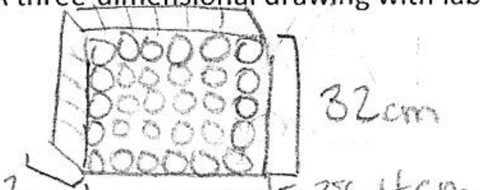
After the goal of the modeling process has been defined, the modeler must identify important factors that control the chosen criterion, then conduct a mathematical investigation. For example, a modeler might identify temperature as a factor that controls the chirp rate of crickets and choose to ignore other factors until the effect of temperature is understood. If accurate predictions of the chirp rate of crickets can be made from temperature alone, no other factors need to be considered.

Your task here is to consider the number of cans in the container as a factor that controls a criterion for optimal packaging. To assess the effect of this factor, no others should be considered, that is, only the number of cans will vary.

Design a secondary package in which the number of cans is not twelve. Remember, everything else should remain the same. Do not change the size or the shape of the can. Do not change the shape of the package – keep it rectangular.

Make a poster and including the following....

1. A three-dimensional drawing with labels of your package.



2. The percent of packaging space used by the cans in your design. Show your work here.

32x38.4x12=14745.6

3.14(3.2²)(12)(30)

11575.3

11575.3 / 14745.6 · 100 = 78.5%

3. The amount of packaging material used per can in your design. Show your work here.

2(1228.8)+140.8(12)

2457.6 + 1689.6

4147.2/30

138.24 cm² per can

4. Group Names and Product Name

Prism Pop

Jordan, Kylee, Lydia

After group presentations, answer the following questions in your groups...

1. Compare the percent of packaging space used by the cans in your design with that used by other designs in class. Is the number of cans a controlling factor if the criterion is maximizing the percent of packaging space used by the cans? Use complete sentences in your answer.

No, the percentage of space used is not dependent upon the size of the box. No matter what size the box is if they are all rectangular prisms the space used will be 78.5%

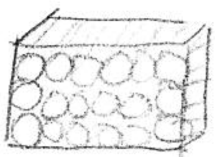
2. Compare the amount of packaging material used per can in your design with the amounts used in other designs. Is the number of cans a controlling factor if the criterion is minimizing the amount of packaging material used per can? Use complete sentences in your answer.

Yes, the number of cans is an important factor because the more cans you put in one box the less packaging will be used per can.

3. Describe the effect of the number of cans as a controlling factor. Interpret the mathematical results that your group has produced, and recommend a package design. Use complete sentences in your answer.

Rectangular prism boxes will be more effective when a higher number of cans is placed in each box because their will be less packaging per can I would reccomend a design with twenty to thirty cans because these higher number will be more effective than twelve, but still relatively easy to carry.

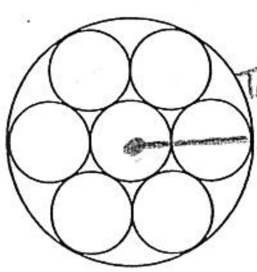
4. Results produced by the mathematical-modeling process must be tested against reality. In the example of soft-drink packaging, the model may not address such considerations as convenience. For example, if you concluded that a package should contain a relatively large number of cans, some consumers may find the package too heavy or think that it requires too much storage space. Use your modeling results to develop a recommendation for a soft-drink company that has decided that a rectangular eighteen-pack is the most convenient. Use complete sentences in your answer.



A 6x3 stack would be the best way to organize a box of 18 cans. It will be the easiest to carry and to fit in fridges due to its combination of the lowest length and height possible.

5. *You modeling efforts have been successful when the efficiency criterion Is minimizing the packaging material used per can. But you have been unsuccessful in improving efficiency when the criterion is maximizing the package space used by the cans. When the mathematical-modeling process fails to produce results, part of the process must be repeated. You next examine another potential controlling factor: package shape.*

Shown here is a cylindrical seven-pack. Determine the percent of package space used by the cans. How does this design compare with the standard twelve-pack? Show your work and use complete sentences in your answer.



Standard 12-pack uses = 78.5% of the space

The Cylindrical pack of seven cans uses 77.7 % of the space.

$3.14(3.2^2)(12)(7) = 2700.9024$

$3.14(9.6^2)(12) = 3472.5888$

$2700.9024 / 3472.5888 = .7777$

77.7%